Hierarchical Anderson model

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Our goal is the spectral analysis of hierarchical Schrödinger operator $H = -L + V$, where L is a hierarchical Laplacian and V is a potential. In the central part of this talk we will consider the random potentials of the special class, i.e. Hamiltonian H will be a hierarchical generalization of the classical Anderson model, where $H = \Delta + \sigma V(x, \omega), x \in Z^d, \Delta$ is the lattice Laplacian and $V(x, \omega)$ are i.i.d.r.v. The coupling constant σ is the measure of the disorder.
We will work with the simplest model, so-called Dyson dyadic model, and realize the hierarchical Laplacian L in the space $L^2([0, \infty), dx)$.
The half-axis $X = [0, \infty)$ in this case is equipped by the special metric which transforms X into ultra-metric space. The study of this metric is based on the hierarchical family of the partitions $\Pi(r), r = 0, \pm 1, \pm 2, \ldots$ of X onto intervals of the rank r: $I(r, i) = [(i-1)2^r, i2^r], i = 1,2, \ldots$. The Laplacian L is the appropriate linear combination of the elementary operators which essentially are the averagings over intervals $I(r, i)$.

Operator $-L$ is an invariant with respect to the renorm – group associated to the family of the partitions $\Pi(r)$. Its spectrum has a well-known structure: discrete set of eigenvalues of the infinite multiplicity with the accumulation at $\lambda = 0$. The eigenvalues and eigenfunctions provide the simple construction of the Markov semi-group $\exp(tL)$ and the natural definition of the (spectral) dimension s of the ultra-metric space X.
Spectral theorem. Let's fix some rank r_0 and put $V(x, \omega) = \sigma X_i, x \in I(r_0, i)$. Here X_i are i.i.d.r.v. with bounded distribution density $p(v), \sigma > 0$ is a coupling constant . We call such $V(\cdot)$ the Anderson potential of the rank r_0 . Then for the arbitrary dimension $s > 0$ and arbitrary level σ of the disorder, the random operator $H = -L + \sigma V(x, \omega)$ has P – a.s. pure point spectrum (complete localization).